Discussion

Comments on "On estimating the Weibull modulus for a brittle material"

It has come to our attention that Equation 3 of the paper by Trustrum and Jayatilaka [1], which is supposed to be valid for tensile strength under uniform loading, is employed by them for analysing the data of a set of experimentally observed flexural strengths, tested in three-point bending. In fact, by using the Weibull function of specific risk of fracture:

$$\phi(\sigma) = \left(\frac{\sigma - \sigma_1}{\sigma_0}\right)^m \qquad \sigma_1 < \sigma < \infty,$$

and

$$\phi(\sigma) = 0 \qquad 0 < \sigma < \sigma_1, \tag{1}$$

where σ is the tensile strength, σ_0, σ_1 and m are the Weibull constants, the correct expression for three-point bending, after a paper [2] of one of the authors, was shown to be

$$F(\sigma) = 1 - \exp\left[-\frac{\sigma_1^{m+1}\, bhL}{V_0\sigma_0^m 2(m+1)\sigma} \times \int_1^{\sigma/\sigma_1} \frac{(\eta-1)^{m+1}}{\eta}\, d\eta\right], \tag{2}$$

where $F(\sigma)$ is the cummulative probability of failure of a rectangular bar of length L, height h and width b subjected to a maximum stress of $\sigma = 3/2\ PL/bh^2$ undergone by the body at fracture under a load, P, at the centre and η is an auxiliary variable under the symbol of the integral. Obviously Equation 2 of the present work is difficult to use in order to evaluate σ_0, σ_1 and m. However if the analytical form of $\phi(\sigma)$ is not specified, an integral equation which when solved allows $\phi(\sigma)$ to be expressed as a function of $F(\sigma)$, was established and solved in [2], to give

$$\phi(\sigma) = \frac{2V_0}{bhL}\frac{d}{d\sigma}\left\{\sigma\frac{d}{d\sigma}\left[\sigma ln\,[1 - F(\sigma)]^{-1}\right]\right\}. \tag{3}$$

When Equation 2 is substituted into Equation 3, the Weibull function is obtained, which is also a proof that Equation 3 is valid.

The difference between the Trustrum and Jayatilaka treatment, with that of the present work can be readily evaluated. In fact, it is easy to show that Equation 3 of [1] excepting a constant, can be written in the form

$$F_T(\sigma) = 1 - \exp\left[-\frac{bhL\sigma_1^m}{2(m+1)^2 V_0\sigma_0^m}\left(\frac{\sigma}{\sigma_1}\right)^m \times \frac{1}{(\sigma/\sigma_1)^m}\left(\frac{\sigma}{\sigma_1} - 1\right)^m\right]. \tag{4}$$

Rearranging Equation 2, with the aim of comparing it with Equation 4 yields

$$F_K(\sigma) = 1 - \exp\left[-\frac{bhL\sigma_1^m}{2(m+1)^2 V_0\sigma_0^m}\left(\frac{\sigma}{\sigma_1}\right)^m \times \frac{m+1}{(\sigma/\sigma_1)^{m+1}}\int_1^{\sigma/\sigma_1}\frac{(\eta-1)^{m+1}}{\eta}\, d\eta\right]. \tag{5}$$

In Equation 4 and 5 the subscripts T and K refer to the work of Trustrum and Jayatilaka [1] and the present authors respectively. By transforming Equation 4 and 5 conveniently, in order to perform a Weibull plot, one obtains

$$\ln\left\{\ln\left[\frac{1}{1 - F_T(\sigma)}\right]\right\} = \ln\left[\frac{bhL\sigma_1^m}{2(m+1)^2 V_0\sigma_0^m}\right] + m\ln(\sigma/\sigma_1) + \ln\left\{\frac{1}{(\sigma/\sigma_1)^m}\left(\frac{\sigma}{\sigma_1} - 1\right)^m\right\}, \tag{6}$$

and

$$\ln\left\{\ln\left[\frac{1}{1 - F_K(T)}\right]\right\} = \ln\left[\frac{bhL\sigma_1^m}{2(m+1)^2 V_0\sigma_0^m}\right] + m\ln(\sigma/\sigma_1) + \ln\left\{\frac{m+1}{(\sigma/\sigma_1)^{m+1}}\int_1^{\sigma/\sigma_1}\frac{(\eta-1)^{m+1}}{\eta}\, d\eta\right\}. \tag{7}$$

The right-hand side terms of Equation 6 and 7 have the same limiting values at infinity, that is,

$$\lim_{\sigma/\sigma_1 \to \infty} \frac{1}{(\sigma/\sigma_1)^m}\left(\frac{\sigma}{\sigma_1} - 1\right)^m$$

$$= \lim_{\sigma/\sigma_1 \to \infty} \frac{m+1}{(\sigma/\sigma_1)^{m+1}}$$

$$\times \int_1^{\sigma/\sigma_1} \frac{(\eta - 1)^{m+1}}{\eta}\, d\eta = 1 \qquad (8)$$

and also in $\sigma/\sigma_1 = 1$, because both terms of expression 8 vanish at this value. Nevertheless for $1 < \sigma/\sigma_1 < \infty$, and $m = 5$, the differences between the left-hand side terms of Equation 6 and 7 differ by a factor of two or even more, as it was concluded from a Weibull plot not shown here. Thus, the Trustrum and Jayatilaka treatment is not satisfactory for three-point bending, but can be used in tensile strength under uniform loading.

References

1. K. TRUSTRUM and A. DE S. JAYATILAKA, *J. Mater. Sci.* **14** (1979) 1080.
2. P. KITTL, *Res. Mechanica* **1** (1980) 161.

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